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## Magnetic properties and critical behaviour of amorphous $\text{RE}_{40}\text{Y}_{23}\text{Cu}_{37}$ ( $\text{RE} = \text{Tb}, \text{Dy}, \text{Ho}$ and $\text{Er}$ ) random anisotropy magnets

A del Moral†, J I Arnaudas†, B D Rainford‡ and C Cornelius‡

† Laboratorio de Magnetismo, Departamento de Física de la Materia Condensada & Instituto de Ciencia de Materiales de Aragón (ICMA), Universidad de Zaragoza & CSIC, 50009-Zaragoza, Spain

‡ Department of Physics, University of Southampton, Southampton SO9 5NH, UK

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**Abstract.** Magnetization measurements have been performed on the amorphous alloys  $\text{RE}_{40}\text{Y}_{23}\text{Cu}_{37}$  ( $\text{RE} = \text{Tb}, \text{Dy}, \text{Ho}$  and  $\text{Er}$ ) over a wide range of magnetic fields (0 to 7 Tesla) and temperatures. These systems show no spontaneous magnetization and are characterized as disordered magnets with random magnetic anisotropy (RMA). Although the low field susceptibility does not diverge it is close to the demagnetizing limit for  $T \leq T_{\text{SG}}$ . The ratio  $D/J$  of the strength of the RMA axial crystal field parameter  $D$  to the ferromagnetic exchange parameter  $J$ , is large for these systems. We have performed a ferromagnetic-like scaling analysis of the magnetization data for large applied fields, thereby determining the critical RMA exponents,  $\beta_a$ ,  $\gamma_a$  and  $\delta_a$ . The non-linear susceptibility,  $\chi_{\text{nl}}$ , is taken to be the order parameter, as for canonical spin glasses;  $\chi_{\text{nl}}$  also shows critical behaviour. Both of these results point to a phase transition at  $T_{\text{SG}}$ . The thermal variation of the high-field magnetization and also of the magnetostriction support this conclusion, as does the existence of transition lines of the de Almeida–Thouless type.

### 1. Introduction

The random magnetic anisotropy problem has been attracting attention since the early work of Harris *et al* (1973) (Moorjani and Coey 1984). Randomness in the magnetic anisotropy can arise from randomness in the local crystalline electric field (CEF), as is found, for example, for non-S-state rare-earth ions in an amorphous environment. In an amorphous rare-earth alloy, the fluctuations in the local atomic coordination will lead to randomness not only in the magnitude of the anisotropy, but also in the orientation of the local easy axes of magnetization. This disorder has a profound effect on the magnetic properties of such systems. In particular it is predicted that a ferromagnet with random magnetic anisotropy (RMA) will not possess long-range magnetic order, no matter how weak the anisotropy (Imry and Ma 1975, Aharony and Pytte 1980). Strong RMA is expected to produce a highly disordered ground state, with a freezing of the spin directions below a certain temperature  $T_{\text{SG}}$ , similar to that observed in random exchange spin glasses (SG) (Chen and Lubensky 1977, del Moral *et al* 1986, del Moral and Arnaudas 1989). Aharony and Pytte (1980) (henceforward AP) predicted that the RMA phase would possess an infinite initial susceptibility,  $\chi_0$ ,

at  $T_{SG}$  and below; however, calculations made to higher order in the expansion of the transverse susceptibility in powers of  $D/J$  (where  $D$  is the average strength of the local axial CEF and  $J$  the exchange interaction) suggest that  $\chi_0 \simeq (J/D)^4$  (Chudnovsky *et al* 1986). In addition, for RMA systems with ferromagnetic exchange interactions,  $D/J$  is an important parameter because the magnitude of this ratio indicates whether we are dealing with strong or weak RMA, and hence whether our systems are respectively of SG type (strongly disordered), or 'coherent spin glasses' (CSG), a phrase used by Chudnovsky *et al* (1986) to describe a quasi-ferromagnetic state in which the magnetization direction wanders gradually.

In this paper, we present evidence that the amorphous rare-earth alloys  $RE_{40}Y_{23}Cu_{37}$  (with RE a non-S-state rare-earth ion) are indeed systems possessing strong RMA, with many of the expected features, including cusps in the initial susceptibility at  $T_{SG}$ , coercivity and hysteretic behaviour below  $T_{SG}$  and magnetic after-effects at remanence (Moorjani and Coey 1984). Evidence is produced that the spin freezing at  $T_{SG}$  may be a true phase transition, as recently observed in the diluted crystalline Laves phase compounds  $Dy_xY_{1-x}Al_2$  (del Moral *et al* 1988).

The organization of the paper is the following: in section 2 we present the experimental details; in section 3 we deal with the features of the magnetization measurements typical of RMA systems; in section 4 we perform a ferromagnetic-like scaling analysis in order to relate the RMA scaling exponents with those of a pure ferromagnet (Gehring *et al* 1990); in section 5, we deal with the non-linear susceptibility. Finally, in section 6, we discuss our findings and extract conclusions.

## 2. Experimental details

We will describe the experimental details briefly, since these have already been discussed in detail elsewhere (del Moral and Arnaudus 1989). The amorphous ribbons of  $RE_{40}Y_{23}Cu_{37}$  compounds (RE = Tb, Dy, Ho and Er) were prepared by melt spinning. Magnetization isotherms, in the temperature range from 4.2 K up to well above the freezing temperatures,  $T_{SG}$ , were measured with a Faraday force magnetometer. This had superconducting solenoids allowing applied magnetic fields up to 7 T, with a maximum field gradient of  $\cong 0.1 \text{ T cm}^{-1}$ .

## 3. Magnetization, Arrott plots and the approach to saturation

In figure 1 we show, as an example, the magnetization isotherms for the  $Tb_{40}Y_{23}Cu_{37}$  compound. Even at 7 T these are far from saturation, and the strong curvature of  $M(H)$  is preserved even well above  $T_{SG}$  (see table 2 and del Moral *et al* 1986). The same behaviour is found for the other measured  $RE_{40}Y_{23}Cu_{37}$  alloys (RE = Dy, Ho and Er). In the following we describe the detailed features of the magnetization data relevant to our later discussion.

First of all, the plots of  $M$  versus  $T$  at low magnetic fields (between around 0.1 and 3 T) display broad cusps (see figure 2 for the Tb alloy). These cusps shift to lower temperatures with increasing applied magnetic field. In figure 3 we show plots of the field versus the reduced cusp temperature,  $t_H = [T_{SG}(0) - T_{SG}(H)]/T_{SG}(0)$ , where  $T_{SG}(0) = T_{SG}$  is the transition temperature previously defined. They are found to follow the law

$$H = H_0 t_H^{\phi/2}. \quad (1)$$

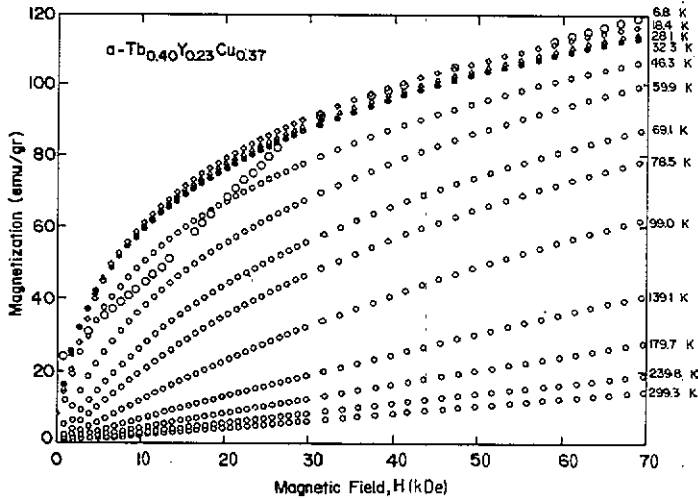


Figure 1. Isotherms of magnetization versus magnetic field for the alloy  $a\text{-Tb}_{40}\text{Y}_{23}\text{Cu}_{37}$ .

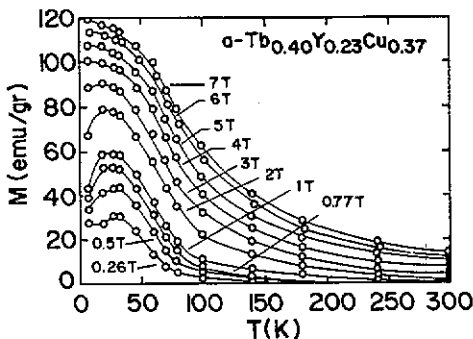
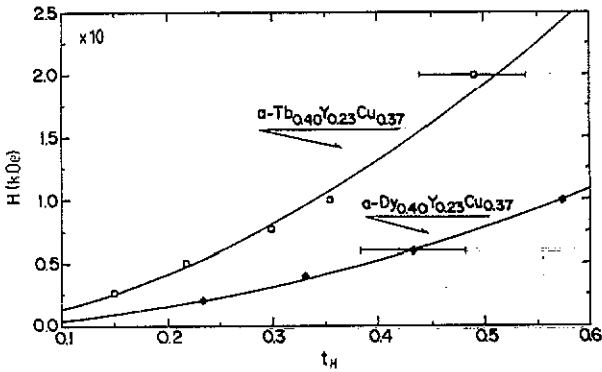


Figure 2. Thermal variation of magnetization at increasing magnetic field for the alloy  $a\text{-Tb}_{40}\text{Y}_{23}\text{Cu}_{37}$ . The continuous lines are guides to the eye.

Fischer and Zippelius (1986) showed that a crossover line of this form exists for the limit of large RMA (Ising limit), where the system crosses from the SG state to one with the spins aligned along the magnetic field. This line is of the same form as the de Almeida–Thouless line (de Almeida and Thouless 1978) for the random exchange Ising spin glass, for which the crossover exponent  $\phi_{AT} = 3.0$ . For the present results we find a value of  $\phi = 3.5$ , not far from  $\phi_{AT}$  (see table 1, where the values of the characteristic field  $H_0$  are included).

We showed previously (del Moral and Arnaudas 1989) that a simultaneous determination of the RMA SG order parameter, the field-induced magnetization  $M$  and the anisotropic magnetostriction  $\lambda$  allow  $D$  and  $J$  to be determined separately. In particular, our RMA replica model (del Moral and Arnaudas 1989) gives the same result for the induced magnetization in the RMA SG as the Sherrington–Kirkpatrick



**Figure 3.** Field lines at the cusps of the magnetization versus the reduced temperature,  $t_H = [T_{SG}(0) - T_{SG}(H)]/T_{SG}(0)$ , for the alloys  $\alpha$ -Tb<sub>40</sub>Y<sub>23</sub>Cu<sub>37</sub> and  $\alpha$ -Dy<sub>40</sub>Y<sub>23</sub>Cu<sub>37</sub>.  $T_{SG}(0)$  is the zero-field SG temperature. The continuous lines are the theoretical fits to the de Almeida-Thouless lines (equation (1)) with crossover exponents  $\phi$  as given in table 1.

**Table 1.** Scaling exponents for the  $\alpha$ -RE<sub>40</sub>Y<sub>23</sub>Cu<sub>37</sub> systems using a ferromagnetic-like scaling analysis. The  $\gamma_a$  exponent was obtained using the scaling relation,  $\gamma_a = \beta_a(\delta_a - 1)$ ;  $\phi$  is the crossover exponent for the instability of the SG state; the characteristic field  $H_0$  (see (1)) is also included.

Compound	$\beta_a$	$\delta_a^a$	$\delta_a^b$	$\gamma_a$	$\delta_1$	$\phi$	$H_0$ (T)
Tb <sub>40</sub> Y <sub>23</sub> Cu <sub>37</sub>	0.54	3.31	3.03	1.25	2.5	3.4	6.18
Dy <sub>40</sub> Y <sub>23</sub> Cu <sub>37</sub>	0.60	2.70	2.77	1.0	1.4	3.5	2.69
Er <sub>40</sub> Y <sub>23</sub> Cu <sub>37</sub>	0.50	2.80	2.80	0.90	2.1	—	—

<sup>a</sup> Obtained from the 'collapse' of data points in a plot  $\ln(M/|t|^{\beta_a})$  versus  $\ln(H/|t|^{\beta_a + \gamma_a})$ .

<sup>b</sup> Obtained from the slopes of the above plots in the region  $t \cong 0$ .

(1975) model for the random exchange Ising spin glass, namely

$$M = \int_{-\infty}^{\infty} \frac{dx}{\sqrt{2\pi}} e^{-x^2/2} \langle S_z \rangle \quad (2)$$

where  $\langle S_z \rangle$  is the canonical thermal average of the spin component along the axis of the applied field. The average over the dummy variable  $x$  is over the RMA spin disorder. The detailed way in which this average is performed is fully described in del Moral and Arnaudas (1989). The variation of the high-field magnetization ( $H = 7$  T) with temperature is very well fitted by (2), indicating an RMA spin-glass character for the present alloys (see figure 4). The values of the  $D$  and  $J$  parameters used for such fittings are collected in table 2. Their large ratios indicate the presence of strong RMA in these alloys.

Finally, we briefly mention the magnetic after-effect observed when leaving the samples at remanence at temperatures below  $T_{SG}$ . In figure 5 we show, for the Dy alloy, the time dependence of the remanent magnetization at 4.2 K, which displays two regimes. We found that the slower one follows a logarithmic law with the time,  $t'$  (Fischer 1985).

$$M_r(t') = M_{0r} - S' \ln t' \quad (2a)$$

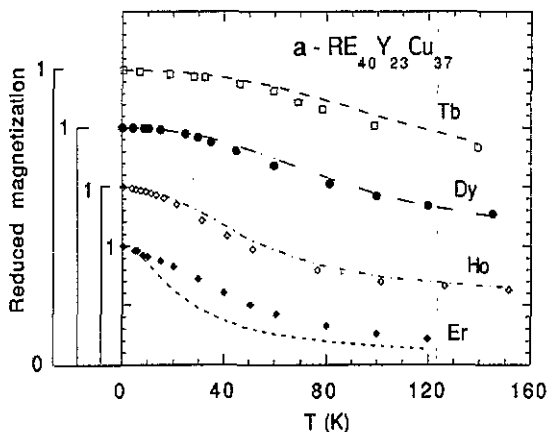


Figure 4. Thermal variation of the reduced magnetization at 7 T for the amorphous  $RE_{40}Y_{23}Cu_{37}$ . The lines are the theoretical replica model prediction (see details in the text).

Table 2. Spin-glass temperature,  $T_{SG}$ , random CEF strength parameter,  $D$ , and exchange constant,  $J$ , for the amorphous  $RE_{40}Y_{23}Cu_{37}$  series of alloys.

Compound	$T_{SG}$ (K) <sup>a</sup>	$D$ (K) <sup>b</sup>	$J$ (K) <sup>b</sup>
$Tb_{40}Y_{23}Cu_{37}$	36.0	+3.0	1.25
$Dy_{40}Y_{23}Cu_{37}$	23.5	+1.25	0.66
$Ho_{30}Y_{23}Cu_{37}$	12.6	+0.60	0.22
$Er_{40}Y_{23}Cu_{37}$	7.0	-0.37	$\sim \sim 0.0$

<sup>a</sup> From del Moral *et al* (1986), AC (15 Hz), low-field ( $\cong 35$  mOe) susceptibility measurements.

<sup>b</sup> From del Moral and Arnaudus (1989).

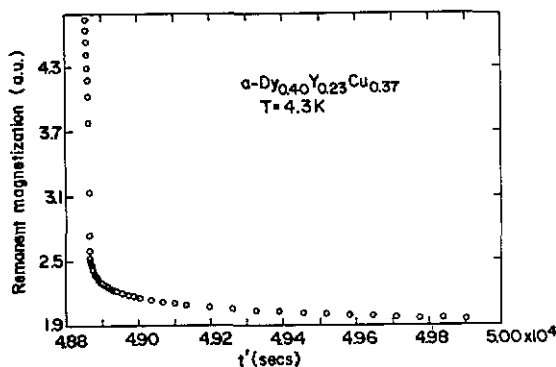


Figure 5. Magnetic after-effect of the isothermal remanent magnetization at 4.3 K for the alloy  $a-Dy_{0.40}Y_{0.23}Cu_{0.37}$ .

with the viscosity parameter  $S'$  being very similar for the different alloys. Assuming a distribution of energy barriers among the spin replicas with constant amplitude up to a maximum cut-off energy,  $E_0$ , it can be shown (Fischer 1985) that  $S' = k_B T / E_0$ .

The similarity in the values of  $S'$  means that our alloys, with different RE ions, possess similar values of  $E_0$ . A likely explanation of this could be the similarity, for the different alloys studied, of the products  $E_0 = K_{\parallel} V$  (Chudnovsky and Gunther 1988), where  $K_{\parallel}$  is the average uniaxial macroscopic anisotropy for a spin cluster of average volume  $V$ .

### 3.1. Arrott plots

Except for in the case of a vanishingly small  $D/J$ , the equation of state for RMA systems (AP 1980, Goldschmidt and Aharony 1985, Gehring *et al* 1990) is far more complex than for usual ferromagnets. However, the ferromagnetic-like critical exponents  $\beta_a$  and  $\gamma_a$  obtained for these compounds (see section 4 below) are not very far from the values 1/2 and 1 respectively, and therefore a plot of  $M^2$  versus  $H/M$  is justified in order to ascertain whether or not these amorphous  $\text{RE}_{40}\text{Y}_{23}\text{Cu}_{37}$  systems possess a spontaneous magnetization and whether the low-field susceptibility  $\chi_0 \equiv (M/H)_{H \rightarrow 0}$ , diverges at  $T_{\text{SG}}$  and below. In figures 6(a)–(b) we show the Arrott plots for the Dy and Er alloys, with the estimated values of the expected demagnetizing field limits  $(H/M)_{\text{dem}} = 4\pi N$ ,  $N$  being the demagnetizing factor of the sample. There are three main features in such isotherms:

(i) isotherms at and below  $T_{\text{SG}}$  are close to, but do not attain the demagnetizing limit;

(ii) the isotherms have the convex curvature expected from the RMA equation of state (see (5) below); and, most importantly

(iii) since the isotherms extrapolate to the  $H/M$  axis at all temperatures, there is no spontaneous magnetization at or below  $T_{\text{SG}}$ .

These are again clear features pointing to RMA character. We can conclude that although  $\chi_0$  does not diverge it may attain the value predicted by Chudnovsky *et al* (1986), namely  $\chi_0 \sim (J/D)^4$ .

This behaviour of non-S-state amorphous rare-earth alloys should be contrasted with that of the corresponding Gd alloys. The magnetic properties of amorphous  $\text{Gd}_x\text{Y}_{68-x}\text{Cu}_{32}$  alloys have been studied in detail by Nakai *et al* (1988). The amorphous  $\text{Gd}_{68}\text{Cu}_{32}$  sample was found to be a good ferromagnet which saturates readily in a field of 1 T. With increasing dilution by yttrium there is eventually a cross-over to a spin-glass phase for  $x < 37$ , due to the existence of competing ferromagnetic and antiferromagnetic exchange interactions. However, for  $x = 40$ , the concentration which corresponds to the present alloys, the alloy still displays long-range ferromagnetism: there is an intercept on the  $M^2$  axis in an Arrott plot. Since Gd has an  $^8\text{S}_{7/2}$  ground state, the crystal field has no effect, to first order, and so we might expect the magnetic anisotropy to be very small. It is clear, then, that the absence of long-range magnetic order in the non-S-state amorphous alloys is a consequence of random anisotropy.

### 3.2. Approach to saturation

The approach to magnetization saturation of RMA magnets is predicted to show quite a distinctive magnetic-field dependence. Theoretical models (Chudnovsky *et al* 1986, del Moral and Cullen 1990) predict a field dependence of the form

$$M(H) = M_s \left( 1 - \frac{1}{15} \sqrt{H_s / (H + H_c)} \right) + \chi_P H \quad (3)$$

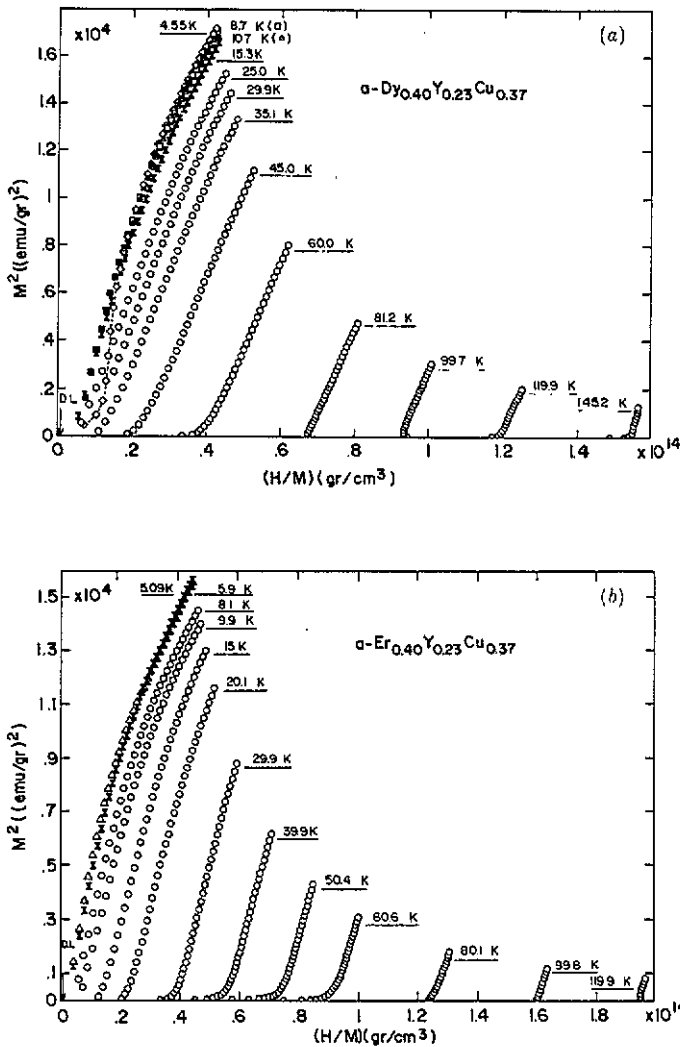


Figure 6. Arrott plots of  $M^2$  versus  $H/M$  for (a) the alloy  $a\text{-Dy}_{0.40}\text{Y}_{0.23}\text{Cu}_{0.37}$  and (b) the alloy  $a\text{-Er}_{0.40}\text{Y}_{0.23}\text{Cu}_{0.37}$ . The vertical arrow marks the demagnetizing field limit value.

where  $\chi_p$  is the Pauli susceptibility,  $M_s$  is the saturation magnetization and  $H_c$  the in-plane coherent anisotropy field parallel to the applied field. This coherent or macroscopic anisotropy most likely has its origin in the magnetoelastic energy associated with the residual stresses, originating from the ribbon preparation procedure. Such an effect is expected to be non-negligible for materials with high magnetostriction such as the present amorphous alloys.  $\chi_p$  is the conduction electron susceptibility. A spin-wave model (del Moral and Cullen 1990) gives for the characteristic field,  $H_s$ ,

$$H_s = (1/g\mu_B)((g-1)^2 v_a J_{RE}^2 / 4\pi)^2 (D^2/A^{3/2})^2. \tag{4}$$

In this expression,  $J_{RE}$  is the total angular quantum number for the RE 4f shell,  $v_a$  the atomic volume per spin and  $A$  the spin-wave stiffness constant,  $A = \frac{1}{2} J S a_{nn}^2$ ,  $S$  being the spin quantum number and  $a_{nn}$  the average nearest neighbour distance.



Such an approximation to saturation should be valid in the so-called 'ferromagnet of wandering axis' (FWA) regime, where  $H \geq H_s$  (Chudnovsky et al 1986). In figure 7 we show the fit to (3) of some low-temperature isotherms for different alloys, and the values of  $M_s$ ,  $H_s$  and  $\chi_P$  used for the fits are collected in table 3. Estimates of  $H_s$  using (4) and the  $D$  and  $J$  values from table 2, are also given in table 3. The conclusion is that the form of (3) is extremely well obeyed in the whole range of fields explored, another characteristic of RMA. The values of the coherent anisotropy field  $H_c$  are seen to be small, although not negligible. The values of  $H_s$ , however, are around two orders of magnitude greater than those estimated from (4) and, more importantly, much higher than the maximum applied field of 7 T. This problem must reside in the prefactor  $\frac{1}{15}$  in (3), which arises from the assumption of an isotropic distribution of local easy axes, with a zero structural correlation length between them (del Moral and Cullen 1990). To obtain values of  $H_s$  in better agreement with (4), this prefactor should be one order of magnitude bigger.

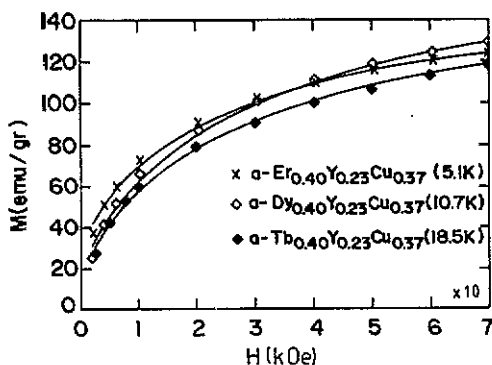


Figure 7. Low-temperature isotherms of magnetization for the a-RE<sub>40</sub>Y<sub>23</sub>Cu<sub>37</sub> series. The continuous lines are theoretical fits with (3) and the parameters  $H_s$ ,  $H_c$  and  $\chi_P$  shown in table 3.

Table 3. Values of  $M_s$  (saturation magnetization),  $H_s$  (characteristic field),  $H_c$  (coherent anisotropy field) and  $\chi_P$  (Pauli susceptibility) derived for the fitting of the isotherms with the theoretical prediction (3) of the approach to saturation. The estimated fitting errors are  $\approx \pm 2\%$  for  $M_s$ ,  $\approx \pm 1\%$  for  $H_s$  and  $\approx \pm 5\%$  for  $H_c$ .

RE	T(K)	$M_s$ (emu g <sup>-1</sup> )	$H_s$ (T)	$H_s$ (T) <sup>a</sup>	$H_c$ (T)	$\chi_P$ (emu g <sup>-1</sup> Oe <sup>-1</sup> )
Tb	18.5	185	254	6.3	1.35	$\approx 10^{-5}$ - $10^{-6}$
Dy	10.7	207	260	1.2	1.40	$\approx 10^{-5}$ - $10^{-6}$
Er	5.1	181	185	—	1.20	$\approx 10^{-5}$ - $10^{-6}$

<sup>a</sup> Values estimated from (4).

#### 4. Ferromagnetic-like scaling analysis and the SG phase transition

We now wish to explore whether a true SG phase transition exists at  $T_{SG}$  for the RE<sub>40</sub>Y<sub>23</sub>Cu<sub>37</sub> alloys. To do this we need to consider several preliminary steps. It was

recently recognized (Gehring *et al* 1990) that, under applied magnetic fields, *weak* RMA systems show features reminiscent of the pure ferromagnetic case. This happens, in particular, in relation to their magnetic equation of state. For RMA systems, this equation can be written in a general scaling form, or modified Arrott plot, as (Gehring *et al* 1990, Goldschmidt and Aharony 1985)

$$(H/M)^{1/\gamma} = t + M^{1/\beta} + a_A[(n-1)/n(n+2)](D/J)^2 M^{1/\beta} (H/M)^{-\epsilon/2\gamma} \quad (5)$$

where  $t = (T_{SG} - T)/T_{SG}$ ,  $\epsilon = 4 - d$  and  $n$  is the spin dimensionality. In the limit  $D/J \rightarrow 0$  this equation of state becomes that of a pure ferromagnet, and therefore must give the same scaling forms as those obtained from the standard equation of state for a pure ferromagnet both for  $t = 0$  and for large  $t$ , i.e.

$$M/|t|^\beta = f(H/|t|^{\beta\delta}) \quad (6)$$

where  $f(x)$  is a scaling function. It is important to note that  $\beta$ ,  $\delta$  and  $\gamma$  are the critical exponents for the *pure ferromagnetic* case. In fact, it can be easily shown from (5) that in the limit  $D/J = 0$  and for  $t = 0$ ,  $M \sim H^{1/\delta}$ , while in the paramagnetic regime ( $t > 0$ ) and in the low-field limit,  $M \sim Ht^{-\gamma}$ . These are indeed the scaling forms for the pure ferromagnet obtained from (6).

One can try to apply (6) to the weak RMA systems, but with different exponents,  $\beta_a$ ,  $\delta_a$  (and  $\gamma_a$ ), characteristic of such a system, i.e. perform a ferromagnetic-like scaling analysis. With this in mind one can obtain from (5) the scaling equations for the RMA case, i.e. when  $D/J \neq 0$  (Gehring *et al* 1990). The results obtained are now summarized for  $T = T_{SG}$ ,

$$M \sim H^{1/\delta_a} \quad (7)$$

where

$$\delta_a = 1 + 2\gamma/(\epsilon + 2)\beta. \quad (8)$$

These results provide a link between the RMA and the pure-ferromagnet exponents. Similarly we may look for the low-temperature ( $t \ll 0$ ) scaling behaviour, where we obtained

$$M \sim H^{1/\delta_1}.$$

In this case the low-temperature RMA exponent becomes

$$\delta_1 = 1 + 2\gamma/\beta\epsilon. \quad (9)$$

Even in the limit of large  $D/J$ , for  $T > T_{SG}$  one obtains from (5) the scaling relation  $\chi \sim t^{-\gamma}$ , as for the pure ferromagnet. We need one further equation relating the  $\beta_a$  and  $\beta$  exponents, i.e.

$$\beta_a = [(\epsilon + 2)/2]\beta. \quad (10)$$

However, the absence of a spontaneous magnetization in RMA systems means that  $\beta_a$  does not have the same crucial meaning as has  $\beta$  for pure ferromagnets.

The relations found between the RMA exponents and those for the pure ferromagnets, and the similarity of the scaling equations obtained from (5) and (6), give strong support to the use of (6) for our RMA systems, which would indicate the existence of a true SG phase transition. However, it is important to recognize that the application of a ferromagnetic-like analysis to our *strong* RMA alloys can only be performed for the regime of large applied magnetic fields, where the local RMA fields can be, to some extent, overcome: the system becomes ferromagnetic-like, with some degree of spin alignment along the applied field. Such a ferromagnetic-like scaling analysis was applied to the amorphous  $\text{RE}_{40}\text{Y}_{23}\text{Cu}_{37}$  alloy data, for applied fields in the range of 0.25 to 7 T. The resulting plots for the Tb and Dy alloys are shown in figures 8(a)–(b). The values chosen for  $\beta_a$  and  $\delta_a$  (see table 1) were those giving the best ‘collapses’ of the experimental data points onto the scaling form. The important feature to be observed is that for the branch  $T < T_{\text{SG}}$ , the magnetization is *not* field independent, which would be indicative of a spontaneous magnetization. This is in good accord with the Arrott plots (see section 3.1). Another feature is that the exponents obtained are quite similar over the whole series and are also similar to those obtained for the crystalline, weak RMA systems  $\text{Dy}_x\text{Y}_{1-x}\text{Al}_2$  (Gehring et al 1990).

A calculation of the RMA exponents  $\beta_a$  and  $\delta_a$  from those for the pure ferromagnetic case, in order to test (8) and (10), is not strictly practicable due to our ignorance of the critical exponents for an equivalent pure ferromagnetic phase for the REYCu alloys. However, the values calculated using the theoretical values  $\beta = 0.367$  and  $\gamma = 1.388$  for the pure 3D Heisenberg ferromagnet (Collins 1989) i.e.  $\beta_a = 0.55$ ,  $\delta_a = 3.52$ , are not very far from the determined values, notably for the Tb alloy (see table 1); the same is true for the values of  $\gamma_a$ . The conclusion is, again, that the  $\text{RE}_{40}\text{Y}_{23}\text{Cu}_{37}$  alloys are really RMA systems. Finally, one can easily show that the low-temperature branch of the scaling equation (6) has a slope

$$\partial \ln(M/|t|^{\beta_a}) / \partial \ln(H/|t|^{\beta_a \delta_a}) = 1/\delta_1$$

but the slopes of the plots at the lowest measured temperatures (see figures 8(a)–(b) and table 1) are very far both from the value predicted by (9), i.e.  $\delta_1 = 8.56$ , and from the AP (1980) calculation,  $\delta_1 = 5$ . In the temperatures regime  $T < T_{\text{SG}}$ , one can show that the above slope becomes  $1/\delta_a$  (see figures 8(a)–(b)). Values of  $\delta_a$  obtained in this way, listed separately in table 1, agree reasonably well with those obtained from the collapse of the data points onto the scaling form.

## 5. Non-linear susceptibility

For RMA systems, where there is no spontaneous magnetization, the strong parallels with the archetypal spin glasses make the non-linear susceptibility  $\chi_{\text{nl}}$  the appropriate order parameter (Suzuki 1977). It is likely, therefore, that the scaling equation of Suzuki (1977) for  $\chi_{\text{nl}}$  is applicable to the present results. We define  $\chi_{\text{nl}}$  as follows:  $\chi_{\text{nl}} = (M/H) - (M/H)_{H \rightarrow 0} \equiv \chi - \chi_0$ . The scaling equation for  $\chi_{\text{nl}}$  (Suzuki 1977, Kattori and Suzuki 1985) is

$$\chi_{\text{nl}} = |t|^{\beta_s} (H^2/|t|^{\beta_s + \gamma_s}) \quad (11)$$

where  $\beta_s$  and  $\gamma_s$  are the critical exponents for  $\chi_{\text{nl}}$ .

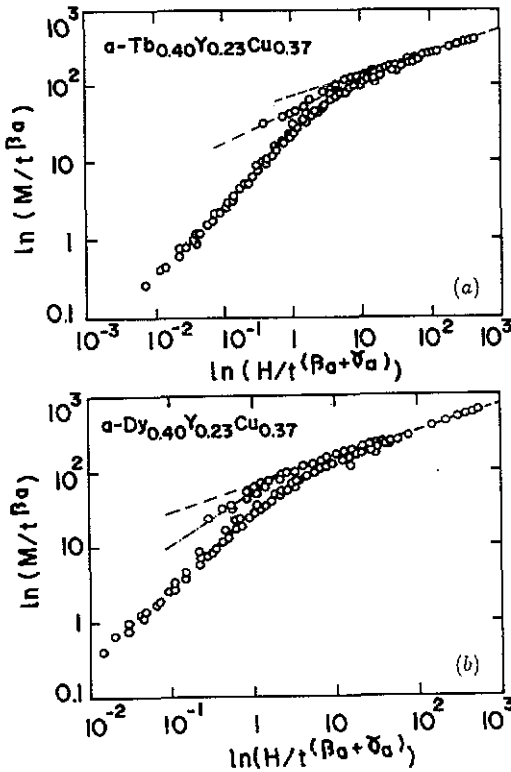
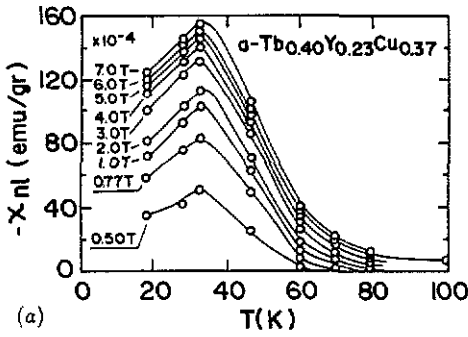


Figure 8. Double-logarithmic scaling of data points for (a)  $a\text{-Tb}_{0.40}\text{Y}_{0.23}\text{Cu}_{0.37}$  and (b)  $a\text{-Dy}_{0.40}\text{Y}_{0.23}\text{Cu}_{0.37}$ , according to the ferromagnetic-like (6) for the reduced temperature intervals  $-0.81 < t < +7.31$  and  $-0.81 < t < +5.18$ , and  $T_{SG} = 36.0$  K and 23.5 K, respectively. The range of applied magnetic field is 0.26 to 7 T. The resulting critical exponents are given in table 1. The upper branch is for  $T < T_{SG}$  and the lower one for  $T > T_{SG}$ . The slope of the discontinuous line gives  $1/\delta_2$  and the slope of the dotted-discontinuous line,  $1/\delta_1$  (see values on table 1).

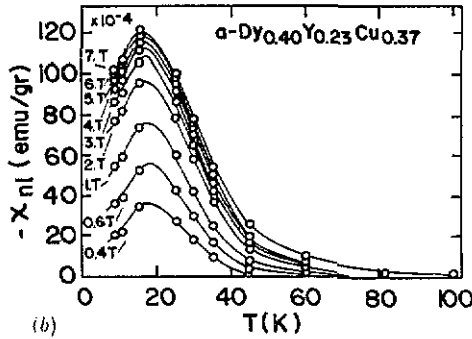
In order to obtain  $\chi_{nl}$  we have arbitrarily taken  $\chi_0$  at the lowest reliable field ( $H \equiv 0.1$  T) which avoided problems with remanence in the superconducting solenoid. Even at 7 T, the systems are far from saturation (see figure 1) and therefore such a field can be considered as being low. In figures 9(a)–(b) we show the thermal variation of  $-\chi_{nl}$  at increasing values of the applied magnetic field. We note that there is no shift of the maximum with the field, as might be expected if  $\chi_{nl}$  is the order parameter, since its maximum value signals the fixed point  $T_{SG}$ ; this points again to a phase transition. Attempts to get good collapse of the data points onto the scaling form of (11) were not wholly successful (see figure 10), probably because the available values of  $t$  were relatively far from  $T_{SG}$ . Moreover, acceptable scaling was obtained only for the  $T < T_{SG}$  branch. Therefore, the values quoted in the captions of figure 10 should be regarded as estimates and not very reliable.

## 6. Conclusions and discussion

All of the features derived from magnetization measurements on the amorphous



(a)



(b)

Figure 9. (a) Non-linear susceptibility versus temperature for increasing applied magnetic field, for (a) the alloy  $\alpha\text{-Tb}_{0.40}\text{Y}_{0.23}\text{Cu}_{0.37}$ , (b) the alloy,  $\alpha\text{-Dy}_{0.40}\text{Y}_{0.23}\text{Cu}_{0.37}$ . The continuous lines are guides to the eye.

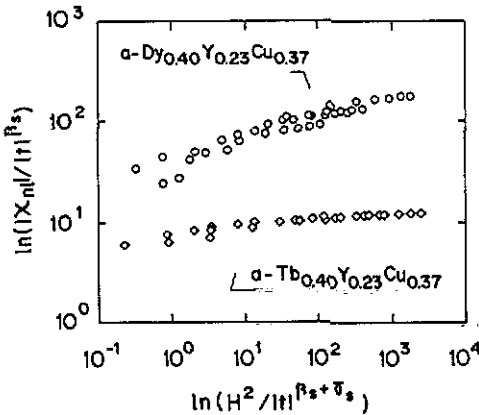


Figure 10. Double-logarithmic plot of the non-linear susceptibility data points, in the form of  $\ln(|\chi_{nl}|/|t|^{\beta_s})$  versus  $\ln(H^2/|t|^{\beta_s+\gamma_s})$ , for the alloys  $\alpha\text{-RE}_{0.40}\text{Y}_{0.23}\text{Cu}_{0.37}$  (RE = Tb:  $-0.10 > t > -0.49$ ,  $7 > H > 0.25$  T; RE = Dy:  $-0.8 > t > -0.35$ ,  $7 > H > 0.4$  T). The estimated values of the critical exponents, giving the better 'collapses' of data points are  $\beta_s \approx 0.5$ ,  $\gamma_s \approx 1.2$  for Tb and  $\beta_s \approx 0.4$ ,  $\gamma_s \approx 3$  for Dy.

series of alloys  $RE_{40}Y_{23}Cu_{37}$  ( $RE = Tb, Dy, Ho$  and  $Er$ ) point to these systems being disordered magnets with a random distribution of magnetic anisotropy axes (RMA). The ratios  $D/J$ , of the magnitude of the anisotropy to the exchange interaction parameters, are quite large (between 1.9 and 2.7), pointing to highly disordered (SG) magnetic ground states for these alloys. These values are considerably larger than those for CSG (quasi-ferromagnetic) systems with weak RMA, such as the crystalline series  $Dy_xY_{1-x}Al_2$ , where the ratio  $D/J \sim 0.05$  (Gehring *et al* 1990). The initial susceptibility  $\chi_0 = (M/H)_{H \rightarrow 0}$  derived from Arrott plots indicates that these systems do not possess a spontaneous magnetization. Moreover, the existence of cusps in the magnetization isofields, which shift to lower temperatures with increasing field (in analogy with the de Almeida-Thouless lines), the thermal variation of the high-field magnetization, the existence of magnetic relaxation at remanence and the critical scaling of the non-linear susceptibility all clearly point to SG character. We should also stress that the thermal variation of magnetostriction is in remarkably good agreement with a previous model of magnetoelastic coupling for RMA spin glasses (del Moral and Arnaudus 1989). Finally, the large  $D/J$  ratios, together with a value of the crossover exponent close to  $\phi_{AT} = 3$ , favours an Ising character for these spin glasses, consistent with the strong random anisotropy limit.

We conclude with some comments about the values obtained for the  $\beta_a$  and  $\delta_a$  exponents. As we have shown, these are quite far removed from those of a pure 3D Heisenberg ferromagnet, indicating that some crossover has taken place when passing to the RMA case. However, these exponents cannot be placed within any known universality class: the conclusion of Ising character for the  $RE_{40}Y_{23}Cu_{37}$  series does not work, inasmuch as a 3D Ising pure ferromagnet has  $\beta = 0.326$  and  $\delta = 4.78$  (Collins 1989). The question then remains as to the precise meaning of these RMA exponents. Further work is clearly required in these systems, e.g. by studying the systematics as a function of the yttrium concentration.

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